2.Resonance in electrical circuits

The purpose of the laboratory studies is experimental verification of basic properties of the serial and parallel resonance in electrical circuits.

2.1. General information

The resonance phenomenon occurs in different physical systems and appears when the system is subjected to periodic stimulation of f_{sr} with frequency equal to own circuit frequency f_o, so $f_o = f_{sr}$.

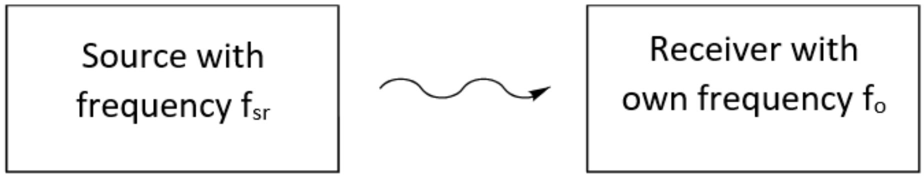

Figure. 2.1. Mechanism of resonance stimulation

In electrical resonance, the frequency of the source is equal to the frequency of its own circuit, which depends only on the value of inductance L and capacitance C. The prerequisite (but not sufficient) for the occurrence of electrical resonance is that the circuit have to consist both elements: capacitors and coils.

Consider the electrical circuit shown in Fig.2.2.

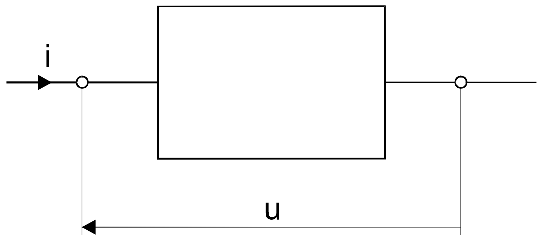

Figure. 2.2. Receiver in electrical circuit

The symbols u, i mean the instant values of the sinusoidal alternating voltage on the receiver and the sinusoidal alternating current in the receiver. The receiver is a serial or parallel combination of R, L, C elements. Using Ohm's law for the value of effective current and voltage can be write:

For serial connection

$$\frac{U}{I} = Z = \sqrt{R^2 + X^2}\ ,\ X = X_L - X_C \qquad (2.1)$$

For parallel connection

$$\frac{I}{U} = Y = \sqrt{G^2 + B^2}\ ,\ B = B_C - B_L \qquad (2.2)$$

where: Z-impedance module, Y-admittance module, R-resistance, X-reactance, G-conductivity, B- susceptance.

The resonance can also be defined as the state of the circuit in which the receiver reactance or susceptance are equal to zero.

If the receiver has a combination of R, C, L, and is a true condition X = 0, there is a **serial resonance**, also known as a **voltage resonance**.

If the receiver has a parallel combination of R, C, L, and is a true condition B = 0, the receiver is in a **parallel resonance**, also known as the **current resonance.**

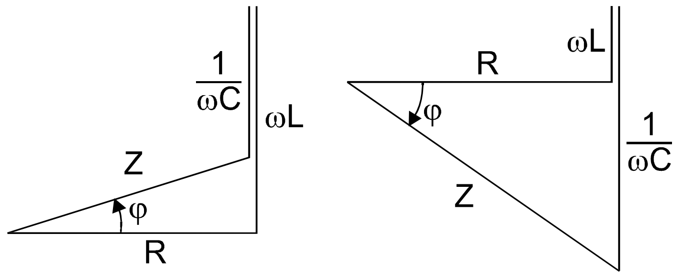

Figure. 2.3. Impedance triangles: a) $\omega L > \frac{1}{\omega C}$; b) $\omega L < \frac{1}{\omega C}$

From the analysis of the resistance triangles shown in fig.2.3 results for the resonance case, i.e. $\omega L = \frac{1}{\omega C}$ true are the dependencies: φ= 0, Z = R, in consequences, the circuit with the resonance does not have a phase shift between the current and the voltage. The circuit behaves as if there is only resistance in it.

In the state of resonance, the active power is:

$$P = U \cdot I \cdot \cos\ \phi = U \cdot I \tag{2.3}$$

the reactive power is:

$$Q = U \cdot I \cdot \sin \phi = 0 \tag{2.4}$$

because **φ= 0**.

This means that all the electricity absorbed by the circuit is transformed into heat in its resistance R. Reactive energy is transferred between elements L and C, omitting the source.

Another definition of electrical resonance is that it is the state of the circuit, in which there is a total internal exchange of reactive energy.

2.1.1. Voltage resonance

Consider a circuit consisting of elements R, L and C connected in series- Fig.2.4.

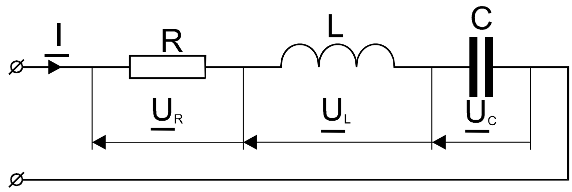

Figure. 2.4. Serial circuit R, L, C

Impedance module Z in this circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{2.1}$$

where: $X_L = \omega L,\ X_C = \dfrac{1}{\omega C}$

and

$$tg\phi = \frac{X_L - X_C}{R} \tag{2.2}$$

Because, at resonance the angle of the phase shift between the current and the voltage of φ= 0 is:

$$tg\varphi = 0,$$

and hence

$$X_L = X_C$$

that is

$$\omega L = \frac{1}{\omega C} \tag{2.5}$$

where ω= 2πf.

The equation (2.5) allows to specify the conditions to be fulfilled in order to be in circuit presented on Fig.2.4 resonance state was occurred.

When the circuit is powered from source with constant frequency f, the resonance state can be obtained by adjusting the L inductance value or the C capacitance (in practice, circuit adjusts to the resonance state by using a capacitor with a regulated capacity).

To achieve resonance state in a circuit with fixed values L and C, use a voltage source with adjustable frequency. The frequency at which resonance occur is called the **resonance frequency** f_R. The f_R frequency value is obtained from the equation (2.5)

$$\omega_r = \frac{1}{\sqrt{LC}} \tag{2.6}$$

or

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{2.7}$$

The phasor graph of the serial circuit in the resonance state shows fig.2.5.

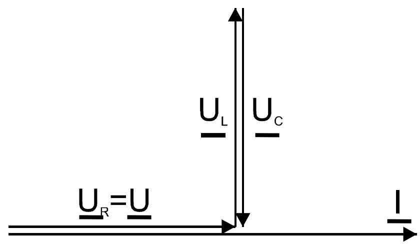

Figure 2.5 Phasor graph of the serial RLC serial circuit in the state of voltage resonance

It should be noted that in the state of the serial resonance, in other words voltage resonance, there is a balancing of voltages on the coil and capacitor $U_L = U_C = 0$. With certain R resistance values, L inductance and C capacitance - voltages U_L and U_C can take relatively high values, although the supply voltage U of the circuit is relatively small. We are saying that there are overvoltages in the circuit .

Characteristic of frequency is plotted to illustrate the resonance circuit properties. These are the characteristics of the current I, the voltage U_L and U_C from the voltage frequency of the power supply. The frequency characteristics of the resonant circuit are derived from the following dependencies:

$$I = \frac{U}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} \tag{2.8}$$

$$U_L = I \cdot X_L \tag{2.9}$$

$$U_C = I \cdot X_C \tag{2.10}$$

In Figure 2.6 the frequency characteristics of the test volumes are shown.

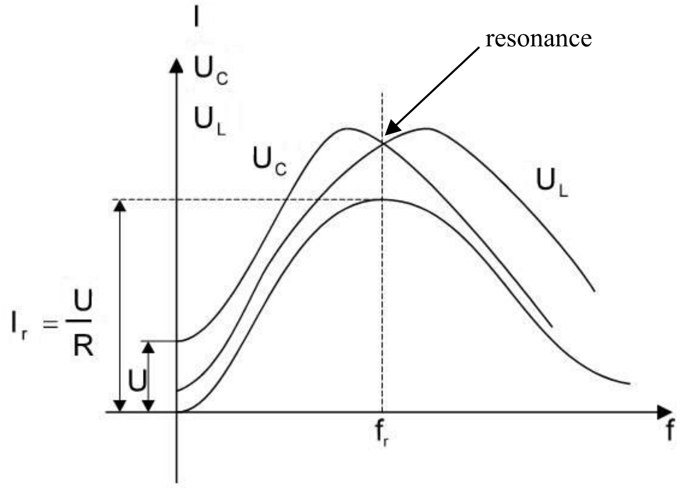

Figure 2.6 Frequency characteristics

As the frequency value increases, the induction reactance X_L rises linearly and the capacitive reactance X_C decreases hyperbolic. At low frequencies, a low-value current is in the circuit, with an angle close to 90° (the circuit has a capacitive character). At large frequencies, a low-value current is delayed by a voltage of at close to 90° (then the circuit has inductive character).

At the resonant frequency $f = f_r$, the reactance values X_L and X_C are equal each other, and the current I achieves the highest value limited only by the resistance R in the circuit ($I_r = \frac{U}{R}$).

Voltage U_C reaches the maximum value for the frequency just before the resonance, whereas the voltage U_L just after the resonance. At the resonant frequency, the voltage U_C and U_L are equal.

Dependence of current I in circuit on frequency f (fig.2.6), it is often called **resonant curve of the circuit**. The shape of this curve depends mainly on the ratio of inductive reactance X_L to resistance R circuit.

This quotient is called the **quality factor** of the circuit:

$$Q = \frac{X_L}{R} = \frac{2\pi fL}{R} \tag{2.11}$$

The quality factor of the circuit is a function of frequency, at the resonant frequency it assumes the value:

$$Q_r = \frac{2\pi f_r L}{R} \tag{2.12}$$

In the resonance state, the voltage on inductance is equal to:

$$U_{Lr} = I_r \cdot X_{Lr} = \frac{U}{R} \cdot X_{Lr} = U \cdot Q_r \tag{2.13}$$

This voltage, equal to the voltage on the capacitor, can be Q_R times greater than the voltage of the power supply. This Q_r-fold increase of voltage on the condenser or coil is a negative phenomenon due to the possibility of a capacitor penetration or coil isolation, while it is the positive phenomenon for many electronic circuits, allowing to generate voltages with specific frequencies. In the radiotechnical circuits Q can take values from 50 to 200.

In Figure 2.7 the same resonant curves of circuits with different values of quality factor were shown.

Such way of presenting resonant curves facilitates significantly analyzing the properties of the resonance circuit. From Figure 2.7 results that the greater is the quality factor of the resonant circuit, the sharpest is the resonant curve. The quality factor of the circuit in principle is determined by the quality of the coil, because in it focus almost all energy losses in the circuit. In a circuit with sufficient quality factor (a dozen and more), even with small detuning from the resonant frequency, the current will rapidly diminishing compared to its resonance value. This means that only sources with frequencies close to the resonant frequency of the circuit can cause the resonant circuit impedance module to be equal to or close to its resistance. Otherwise - the circuit achieves a minimum impedance in a specific frequency band. This is referred to as the loop-through **band**, i.e. the bands - 2Δf, surrounded by the resonant frequency f_R, in which the rms value of the current I in the circuit falls to $\frac{1}{\sqrt{2}} \approx 0{,}707$ value of this current at resonance (see fig.2.7). The transmission width of 2Δf is usually 0.3... 2% of the resonant frequency.

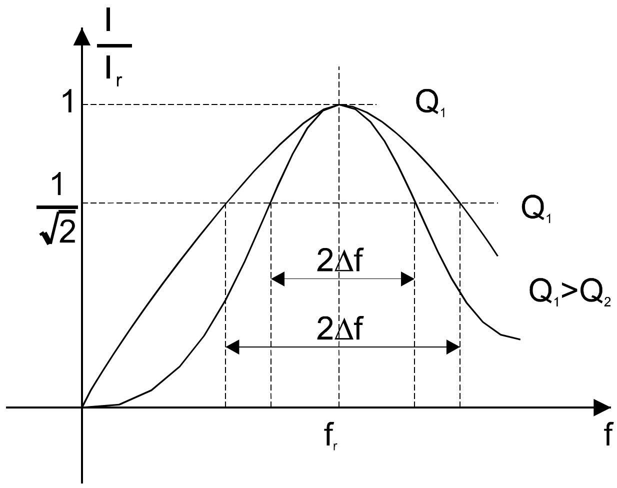

Figure. 2.7 Resonant curves of circuits with different values of quality factor Q

The ability of the circuit to pass currents with frequencies similar to the resonant frequency and practically impermeability of currents of other frequencies is called **circuit selectivity**. The selectivity of the circuit is the greater, when the smaller its transmission band, so the greater its quality factor. The selectivity of the circuit is widely used in radiotechnics.

2.1.2. Currents resonance

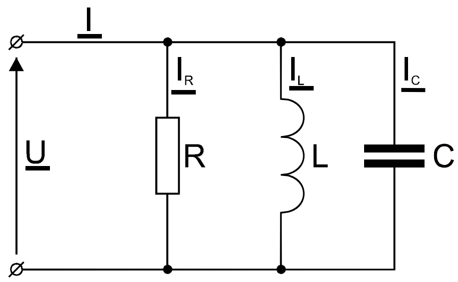

Figure. 2.8 Parallel circuit R, L, C

Consider a circuit consisting of elements R, L, C connected in parallel (fig.2.8)

In Figure 2.9 phasor diagram for this circuit is provided, assuming that there is resonance in the circuit, and therefore the angle of the phase shift between the current I and the U voltage is equal to zero.

In the state of parallel resonance, which is the resonance of currents we have:

$$\underline{I}_L + \underline{I}_C = 0$$

which means that the currents in the coil and capacitor are balanced.

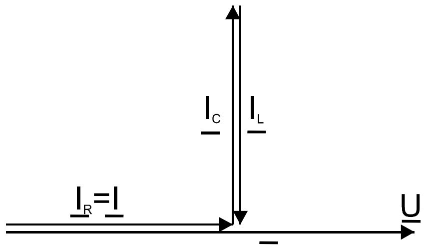

Figure. 2.9 Phasor diagram of R, L, C in current resonance state

Because in the resonance state

$$I_L = I_C$$

and

$$I_L = \frac{U}{X_L}, I_C = \frac{U}{X_C}$$

so

$$X_L = X_C \text{ or } \omega L = \frac{1}{\omega C} \tag{2.14}$$

We have received an expression that must be fulfilled so that the circuit with Fig.2.8 has been in resonance state. The equation (2.14) can be fulfilled by the appropriate selection of inductance L and capacitance C at constant frequency f of voltage source or by changing the frequency of the source when the values of L and C are constant.

From dependent (2.14) we receive the expression on the resonant frequency:

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{2.15}$$

The resonant frequency in this case of current resonance is described by using an identical relationship as the frequency at the serial resonance.

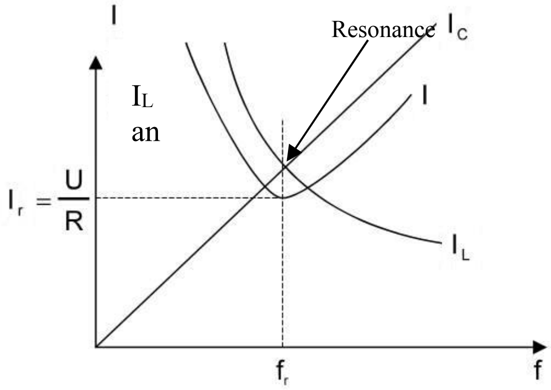

Figure. 2.10 Frequency characteristics

The properties of the parallel resonance circuit are good illustrated by the currents I, I_L, I_C from the source of frequency f

$$I = \sqrt{I_R{}^2 + (I_C - I_L)^2} \tag{2.16}$$

$$I_L = \frac{U}{X_L} = \frac{U}{\omega L} \tag{2.17}$$

$$I_C = \frac{U}{X_C} = U\omega C \tag{2.18}$$

In Figure 2.10 the frequency characteristics of currents I, I_L and I_C were presented. In the resonance state of the current I has a minimum value limited by the resistance R, while the currents I_L i I_C are balanced.
The quality factor of the parallel circuit is essentially related to the loss of power in the capacitor and depends on the ratio of the R resistance to the X_C reactance.
Quality factor $Q = \frac{R}{X_C}$, in the state of resonance:

$$Q_r = \frac{R}{X_{C_r}} = \frac{R}{X_{L_r}} \tag{2.19}$$

and

$$I_{L_r} = I_{C_r} = \frac{U}{X_{L_r}} = \frac{U}{R} Q_r = I_r Q_r \tag{2.20}$$

This means that with the resonance currents I_L i I_C are Q_R times greater than the current being charged by the circuit, what is called the **excess current.**

2.2. Laboratory tests

2.2.1. Voltage resonance

2.2.1.1. Testing the impact of capacitance on voltage resonance

The measuring system is shown in fig.2.11.

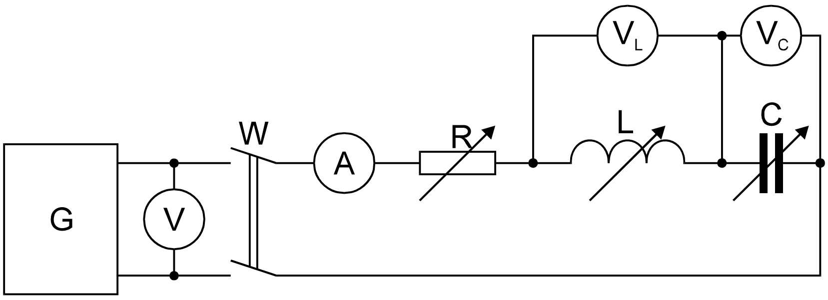

Figure 2.11 Diagram of measuring

Marks:

G - generator, A - ammeter, V_L, V_C, V - voltmeter ,
R - decade resistance, L - decade inductance,
C - decade capacitance.

Measurements for different capacitances are performed in a system, the scheme is given in fig. 2.11. Results of measurements and calculations write in the table 2.1.

Table 2.1. the

U =......, f =......, L =......, R =......

Lp.	Measurement				Calculations	
	C	I	U_L	U_C	X_C	X_L
	μF	mA	V	V	Ω	Ω
1.						
2.						
3.						
4.						
5.						
6.						
7.						

Calculation example:

$X_L=$ $X_C=$

Based on the results of the measurements and calculations, draw graphs I, U_L, U_c in function of X_c. Draw phasor graph for the circuit in the resonance state and for both end-of-the-resonance detuning.

2.2.1.2. Frequency characteristics

Measurements for different frequency values are performed in a system, the diagram of which is given in fig. 2.11. Results of measurements and calculations write in the table 2.2.

Table 2.2. the

U=......, f =. .. ..., L =. .. ..., R =. .. ...

The Lp.	Measurement				Calculations		
	F	I	U_L	U_C	X_L	X_C	$\frac{I}{I_r}$
	Hz	mA	V	V	Ω	Ω	-
1.							
2.							
3.							
4.							
5.							
6.							
7.							

Calculation example:

X_L= X_C= 2Δf = Q_r=

Based on the results of the measurements and calculations, present in a single diagram frequency characteristic of I, U_L, U_C.

Draw resonant curves of circuit in the common chart: I = F(f).

Specify the transmission bandwidth 2Δf and quality factor of the circuit in resonant state.

2.2.2. Current resonance

2.2.2.1. Testing the impact of capacitance on voltage resonance

The measuring system is shown in fig. 2.12.

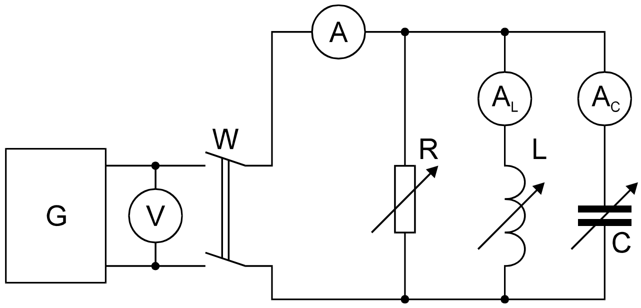

Figure. 2.12. Measuring System diagram

Marks:

G - generator, A, A_L, A_C, - ammeter, V - voltmeter ,
R - decade resistance, L - decade inductance,
C - decade capacitance.

Measurements for different capacitances are performed in a system, the scheme is given in fig. 2.12. Results of measurements and calculations write in the table 2.3.

Table 2.3. the

U =......, f =......, L =......, R =.... ..

Lp.	Measurement				Calculations	
	C	I	I_C	I_L	X_C	X_L
	μF	mA	mA	mA	Ω	Ω
1.						
2.						
3.						
4.						
5.						
6.						
7.						

Calculation example:

X_L= X_C=

Based on the results of the measurements and calculations, draw the graphs I, I_L, I_C in function of X_c. Draw phasor graph for the circuit in the resonance state and for both end-of-the-resonance detuning.

2.2.2.2. Frequency characteristics

Measurements for different frequency values are performed in a system, the diagram of which is given in fig. 2.12. Results of measurements and calculations write in the table 2.4.

Table 2.4. the

U=......, L =….., C =….., R =……

Lp.	Measurement				Calculations	
	F	I	I_C	I_L	X_C	X_L
	μH	mA	Has	Has	Ω	Ω
1.						
2.						
3.						
4.						
5.						
6.						
7.						

Calculation example:

X_L= X_C=

Based on obtained results and calculations present in the diagram frequency characteristics of I, I_C, I_L of the circuit.

2.3. Remarks and conclusions

Compare characteristics obtained in the laboratory studies with the characteristics known from the theory.

Literature

[1] Krakowski M. Theoretical electrical Engineering. Linear and nonlinear circuits. T. I, WNT, 1995

[2] Collective work: Electrical Engineering and electronics for non-electricians, WNT, Warsaw, 1999.